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A COMPARISON OF ADOMIAN DECOMPOSITION METHOD (ADM) AND HOMOTOPY PERTURBATION METHOD (HPM) FOR NONLINEAR PROBLEMS

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ABSTRACT

In this paper, approximated solutions for some non linear problems are calculated by the *Adomian Decomposition Method* (ADM) and *Homotopy Perturbation Method* (HPM) and then the results are compared. The fact that the *Homotopy Perturbation Method* (HPM) solves nonlinear problems without using *Adomian's polynomials* is a clear advantage of this technique over the *Adomian Decomposition Method* (ADM).

KEYWORDS: Adomian Decomposition Method, Homotopy Perturbation Method, System of Nonlinear Differential Equations

INTRODUCTION

In the recent years, for solving a wide range of mathematical, engineering and physical problems, linear and nonlinear, many more of the numerical methods are used. In this paper the *Adomian decomposition Method* (ADM) which was introduced by G. Adomian [5, 6] in the 1980s in order to solve linear and nonlinear functional equations and the *Homotopy Perturbation Method* (HPM), proposed first by Ji-Huan He [8 ,9] in 1998, for solving differential and integral equations, linear and nonlinear, are applied to various problems subject of extensive numerical and analytical studies.

The main idea of this paper is the comparison of the results obtained by *Adomian Decomposition Method* and He's *Homotopy Perturbation Method* while solving some linear and nonlinear partial differential equations. In the following, we will illustrate the ADM introduced by Adomian and also HPM introduced by He.

MAIN IDEA OF ADOMIAN DECOMPOSITION METHOD (ADM)

Consider a non linear differential equation,

$$Ew = g. \tag{1}$$

Here non linear differential operator is E while w and g are the functions of x. In an operator form equation (1) can be written as

$$\Im w + \Re w + \aleph w = g. \tag{2}$$

Here the linear portion of E is represented by an operator $\Im$ which is invertible while $\Re$ is any linear operator which is used for the remainder of linear portion and $\aleph$ is a non linear operator representing the non linear term in E. Introducing the inverse operator $\Im^{-1}$ we get that,

$$\Im^{-1}\Im w = \Im^{-1}g - \Im^{-1}\Re w - \Im^{-1}\aleph w.$$

Since $\Im$ is linear and E is being taken to be a differential operator, $\Im^{-1}$ will represent integration and for any initial conditions provided, $\Im^{-1}\Im w$ will provide an equation for w.Incorporating with the given conditions we get that,

$$w(x) = h(x) - \Im^{-1}\Re - \Im^{-1}\aleph w\,, \tag{3}$$

where $h(x)$ represents a function which is obtained by integrating g, and by using the given initial conditions. The *Adomian Decomposition Method* further decomposes the decomposition into an infinite series of components.

$$w(x) = \sum_{n=0}^{\infty} w_n(x). \tag{4}$$

The non linear term $\aleph\, w$ is equated to an infinite series of polynomials.

$$\aleph(w) = \sum_{n=o}^{\infty} A_n. \tag{5}$$

Where A_n are the *Adomian polynomials* determined by,

$$A_n = \frac{1}{n}\left[\frac{d^n}{d\phi^n}\aleph\, w(\phi)\right]. \tag{6}$$

Putting equations (5) and (6) in equation (2) we have that,

$$\sum_{n=0}^{\infty} wn(x) = w_0 - \Im^{-1}\left(\sum \Re w_n + \sum_{n=0}^{\infty} A_n\right), \tag{7}$$

The recursive relations are founded as, $w_0 = \mathrm{h}(x).$

$$w_{n+1} = \Im^{-1}\Re w_n + \Im^{-1}A_n \tag{8}$$

Having determined the components $w_n,\ n > 0.$ Hence the numerical solution $w,$ follows immediately in the form of series. In order to represent the numerical solution in a closed form the series can also be summed. However for the approximated solution of concrete problems the n-term partial sum will be used.

APPLICATION TO NONLINEAR PROBLEMS

PROBLEM 3. 1 Consider the two dimensional non linear wave equation [12],

$$w_{xx} - ww_{tt} = 1 - \frac{x^2 + t^2}{2}, \tag{9}$$

with initial conditions as,

$$w(x,0) = \frac{t^2}{2}, \quad w_x(0,t) = 0. \tag{10}$$

Rewriting equation (9) in standard form,

$$\Im_x w - \aleph_w = 1 - \frac{x^2 + t^2}{2}. \tag{11}$$

Where, $\Im_x = \frac{\partial^2}{\partial x^2}$, $\Im_w =$ w.$\frac{\partial^2}{\partial t^2}$. If the invertible operator defined by, $\Im^{-1}{}_x = \int_0^x \left(\int_0^x dx \right) dx,$

is applied to equation (11) then we get that,

$$\Im^{-1}{}_x \Im_x w = \Im^{-1}{}_x \left(1 - \frac{x^2 + t^2}{2} \right) + \Im^{-1}{}_x N_w \ . \tag{12}$$

So we get that, $$w = w(0,t) + xw_x(0,t) + \Im^{-1}{}_x \left(1 - \frac{x^2 + t^2}{2} \right) + \Im^{-1}{}_x \aleph_w, \tag{13}$$

And w_0 is defined as, $$w_0 = w(0,t) + xw_x(0,t) + \Im^{-1}{}_x (1 - \frac{x^2 + t^2}{2}) \ . \tag{14}$$

And the non linear term is defined as, $\aleph_w = \sum_{m=0}^{\infty} A_m.$

Here A_m are the *Adomian polynomials*. Then we have, $$w_m = \Im^{-1}{}_x \left(A_{m-1} \right) . \tag{15}$$

Solving equation (14) we get that, $w_0 = \frac{t^2}{2} + \frac{x^2}{2} - \frac{x^4}{24} - \frac{x^2 t^2}{4}.$

Since we have, $$A_0 = w_0 . \frac{\partial^2}{\partial t^2} . w_0 \tag{16}$$

Then we get that, $A_0 = \frac{t^2}{2} + \frac{x^2}{2} - \frac{7x^4}{24} - \frac{x^6}{48} - \frac{x^2 t^2}{2} + \frac{x^4 t^2}{8}.$

Similarly, let us define, $w_1 = \Im^{-1}{}_x \left(A_0 \right) .$

Then we get that, $$w_1 = \frac{x^4}{24} + \frac{7x^6}{720} + \frac{x^8}{2688} + \frac{x^2 t^2}{4} - \frac{x^4 t^2}{24} + \frac{x^6 t^2}{240} \tag{17}$$

As we have that, $A_1 = w_1 . \frac{\partial^2}{\partial t^2} . w_0 +$ w$_0 . \frac{\partial^2}{\partial t^2} . w_1.$ Then we get,

$$A_1 = \frac{7x^4}{24} + \frac{x^2 t^2}{2} - \frac{327x^6}{4320} + \frac{x^6 t^2}{20} + \frac{x^8 t^2}{240} + \frac{1038x^8}{40320} - \frac{99x^{10}}{80640} \ . \tag{18}$$

In the same way if we take, $w_2 = \Im^{-1}{}_x \left(A_1 \right).$ Then we have,

$$w_2 = \frac{7x^6}{720} + \frac{x^4 t^2}{24} - \frac{327x^8}{241920} + \frac{x^8 t^2}{1120} - \frac{x^{10} t^2}{21600} + \frac{1038x^{10}}{36288000} - \frac{99x^{12}}{10644480}. \tag{19}$$

Continuing this process the complete solution is obtained as under,

$w(x,t)=Lim\Phi_m$. By using the relation, $w_{m+1}=\Im^{-1}{}_x\left(A_m\right)$, the N-terms approximation is founded as, $\Phi_m=\sum_{i=0}^{m-1}w_i$.

Following this technique the analytical solution is obtained as,

$$w=\frac{x^2+t^2}{2}.$$

PROBLEM 3. 2 Consider the non linear PDE [1].

$$w_{xx}+ww_x=x+\ln t,\quad t>0. \tag{20}$$

With boundary conditions as

$$w(0,t)=\ln t,\quad w_x(0,t)=1. \tag{21}$$

In an operator form equation (20) can be written as,

$$\Im w=x+\ln t-ww_x, \tag{22}$$

Where $\Im$ is second order partial differential operator such that, $\Im=\frac{\partial^2}{\partial x^2}$. And it is assumed to be invertible and $\Im^{-1}$ is defined as,

$$\Im^{-1}(.)=\int_0^t\int_0^t(.)dtdt. \tag{23}$$

Applying $\Im^{-1}$ to both sides of equation (22) and by using the initial conditions we have that

$$w(x,t)=x+\ln t+\tfrac{1}{6}x^3+\frac{1}{2}x^2\ln t-\Im^{-1}(ww_x). \tag{24}$$

The *Adomian Decomposition Method* defines the solution $w(x)$ in a series form as,

$$w(x,t)=\sum_{m=0}^{\infty}w_n(x,t). \tag{25}$$

Using the decomposed series (25) for $w(x,t)$ in the both sides of equation (24) and by using the recursive relations we have that,

$$w_0(x,t)=x+\ln t, \tag{26}$$

$$w_1(x,t)=\frac{1}{6}x^3+\frac{1}{2}\ln t-\Im^{-1}(w_0w_{0x}). \tag{27}$$

Equations (26) and (27) gives the solution respectively as; $w_0(x,t) = x + \ln t.$ $w_1(x,t) = 0.$ And other consequent components will be, $w_k = 0,\ k \geq 2.$ The exact solution is founded as, $w(x,t) = x + \ln t.$ Which is the required solution of the given problem.

PROBLEM 3. 3 Consider the non linear Klein-Gordon equation [1],

$$w_{tt} - w_{xx} + w^2 = 6xt(x^2 - t^2) + x^6 t^6, \tag{28}$$

with initial conditions as,

$$w(x,0) = 0,\ w_t(x,0) = 0. \tag{29}$$

In an operator form equation (28) can be written as,

$$\Im w = 6xt(x^2 - t^2) + x^6 t^6 + w_{xx} - w^2. \tag{30}$$

Where $\Im$ is second order partial differential operator such that,

$\Im = \dfrac{\partial^2}{\partial x^2}.$ And it is assumed to be invertible and $\Im^{-1}$ is defined as,

$$\Im^{-1}(.) = \int_0^t \int_0^t (.)dtdt. \tag{31}$$

Applying $\Im^{-1}$ to both sides of equation (30) and by using the initial conditions we have that,

$$w(x,t) = x^3 t^3 - \frac{3}{10} x t^5 + \frac{1}{56} x^6 t^8 + \Im^{-1}(w_{xx} - w^2), \tag{32}$$

Using the decomposed series (25) for $w(x,t)$ in the both sides of equation (32) and by using the recursive relations we have that,

$$w_0(x,t) = x^3 t^3. \tag{33}$$

$$w_1(x,t) = -\frac{3}{10} x t^5 + \frac{1}{56} x^6 t^8 + \Im^{-1}(w_{0xx} - w_0{}^2). \tag{34}$$

Equation (34) implies that, $w_1(x,t) = 0.$ As a result of which we have that, $w_k = 0, k \geq 2.$ In a closed form the solution will be,

$$w(x,t) = x^3 t^3.$$

ANALYSIS OF THE PROBLEM WITH HOMOTOPY PERTURBATION METHOD (HPM)

Consider a given non linear PDE,

$$\alpha(w) = h(\Psi); \Psi \in \mho_j, \tag{35}$$

With boundary conditions as,

$$\beta(w, \frac{\partial w}{\partial n}) = 0; \Psi \in \Psi_j, \tag{36}$$

Here α is general differential operator; β is boundary operator, $h(\Psi)$ is any known analytic function boundary of the domain $\mho_j$ is Ψ_j. Then $\mho = \cup \mho_j$, and

$$u_j(\Psi, q) : \mho_j \times [0,1] \to \mathbb{R} \tag{37}$$

The operator α is usually splited into two parts $\Im$ and $\aleph$, where $\Im$ is linear and $\aleph$ is non linear. Therefore equation (35) can be written as,

$$\Im(w) + \aleph(w) = h(\Psi). \tag{38}$$

Let's construct a Homotopy of the type, $u_j(\Psi, q) : \mho_j \times [0,1] \to \mathbb{R},$ which satisfy the equation,

$$\mathrm{H}(u_j, q) = \Im(u_j) - \Im(z_{j,0}) + q\Im(z_{j,0}) + q[\aleph(u_j) - h(\Psi)] = 0. \tag{39}$$

Here embedding parameter $q \in [0,1]$ is, $z_{j,o}$ is the initial approximation of equation (35). So it is obvious that,

$$H(u_j, 0) = \Im(u_j) - \Im(z_{j,0}) = 0, \tag{40}$$

$$H(u_j, 1) = A(u_j) - h(\Psi) = 0, \tag{41}$$

By changing the process from 0 to 1, is just to change that $H(u_j, q)$ from $\Im(u_j) - \Im(z_{j,0})$ To $A(u_j) - h(\Psi)$ In topology such a kind of variation is called deformation. Here $\Im(u_j) - \Im(z_{j,0})$ and $A(u_j) - h(\Psi)$ are called Homotopic. Then the application of perturbation technique, is due to the fact that $o \le h \le 1$, the parameter h is considered as a small parameter, it can also be assumed that the equation (39) will have a solution which can also be expressed in a series form in h as follows,

$$u_j = u_{j,o} + qu_{j,1} + q^2 u_{j,2} + q^3 u_{j,3}. \tag{42}$$

When $h \to 1,$ then equation (39) corresponds to (38) and (42) become the approximated solution of equation (39), i. e.,

$$v = \lim_{h \to 1} u_j = u_{j,0} + u_{j,1} + u_{j,2} + u_{j,3}. \tag{43}$$

The series (43) will be convergent for more cases and the rate of convergence will depend on $A(u_j)$.

APPLICATION TO THE NONLINEAR PROBLEMS

PROBLEM 5.1 Consider the two dimensional non linear wave equations,

$$w_{xx} - ww_{tt} = 1 - \frac{x^2 + t^2}{2}, \tag{44}$$

With initial conditions as,

$w(0,t)=\dfrac{t^2}{2}$, $w_x(0,t)=0$. We construct a *Homotopy* which satisfies,

$$(1-h)\Im[z(x,t;h)-w_0)]+h[\Im z(x,t;h)+zz_{tt}]=1-\frac{x^2-t^2}{2},$$

$$\Im z(x,t;h)-\Im w_0+h\Im w_0+h[zz_{tt}]=1-\frac{x^2-t^2}{2}. \tag{45}$$

With initial approximation as, $z_0(x,t;h)=z(0,t;h)=\dfrac{t^2}{2},\ z_x(x,t;p)=z_x(0,t;h)=0.$

Suppose the solution of equation is of the form, $z(x,t;h)=z_0(x,t)+hz_1(x,t)+h^2z_2(x,t)+...,$ (46)

Then we get,

$$\Im[z_0+hz_1+h^2z_2...]-\Im w_0+h\Im w_0-h[(z_0+hz_1+h^2z_2...)(z_{0xx}+hz_{1xx}+h^2z_{2xx}...)]=1-\frac{x^2-t^2}{2}.$$

With initial approximations as,

$$z(0,t;h)=z_0(0,t)+hz_1(0,t)+h^2z_2(0,t)+...,=\frac{t^2}{2}. \tag{47}$$

$$z_x(0,t;h)=z_{0x}(0,t)+hz_{1x}(0,t)+h^2z_{2x}(0,t)+...,=0. \tag{48}$$

Now the comparing the terms having equal powers of h, we get the following set of equations,

$$\Im z_0=1-\frac{x^2+t^2}{2};z_o(0,t)=\frac{t^2}{2},z_x(0,t)=0. \tag{49}$$

$$\Im z_1+[z_0z_{0xx}]=0;z_1(0,t)=0,z_{1x}(0,t)=0. \tag{50}$$

$$\Im z_2+[z_0z_{1xx}-z_1z_{0xx}]=0;z_2(0,t)=0,z_{2x}(0,t)=0. \tag{51}$$

And so on. Here we note that the same components of the series solution are achieved as we have calculated for the *Adomian polynomials,* in *Adomian Decomposition Method.* When the embedding parameter h goes from 0 to 1, $z(x,t;h)$ goes from the initial approximation to the exact solution as,

$$w(x,t)=\lim_{h\to 1}z(x,t;h)=z_0+z_1+z_2+...,$$

This gives the solutions by using initial conditions as,

$$z_0(x,t)=\frac{x^2+t^2}{2}. \tag{52}$$

$$z_1(x,t)=0. \tag{53}$$

So that we get the solution from equation as,

$$w(x,t) = \frac{x^2 - t^2}{2}. \tag{54}$$

Which is the same solution as it was calculated in *Adomian Decomposition Method*.

PROBLEM 5.2 Consider the non linear PDE,

$$w_{xx} + ww_x = x + \ln t, t > 0, \tag{55}$$

with initial conditions as,

$w(0,t) = \ln t,\ w_x(0,t) = 0.$ We construct a Homotopy which satisfies,

$$(1-h)\Im[w(x,t;h) - w_0)] + h[\Im z(x,t;h) + zz_x] = x + \ln t, \tag{56}$$

$$\Im z(x,t;h) - \Im w_0 + h\Im w_0 + h[zz_x] = x + \ln t, \tag{57}$$

With initial approximations as,

$$w_0(x,t) = w(x,0) = \ln t. \tag{58}$$

Suppose the solution of the form,

$$z(x,t;h) = z_0(x,t) + hz_1(x,t) + h^2 z_2(x,t) + ..., \tag{59}$$

Then we get that,

$$\Im[z_0 + hz_1 + h^2 z_2 + ...] - \Im w_0 + h\Im w_0 + h[(z_0 + hz_1 + h^2 z_2 ...)(z_{0x} + hz_{1x} + h^2 z_{2x} ...) = x + \ln t.$$

With initial conditions as,

$$z(0,t;h) = z_0(0,t) + z_1(0,t) + ... = \ln t, \tag{60}$$

$$z_x(0,t;h) = z_{0x}(0,t) + z_{1x}(0,t) + ... = 1. \tag{61}$$

Now equating the terms with equal powers of h we have ,

$$\Im z_0 = x + \ln t; z_0(0,t) = \ln t, z_{0x}(0,t) = 1. \tag{62}$$

$$\Im z_1 + z_0 z_{0x} = 0; z_1(0,t) = 0, z_{1x}(0,t) = 0. \tag{63}$$

$$\Im z_2 + z_0 z_{1x} + z_1 z_{0x} = 0; z_2(0,t) = 0, z_{2x}(0,t) = 0. \tag{64}$$

And so on. Note that we are getting the same components of the series solution as we have calculated for the *Adomian polynomials*, in *Adomian decomposition method.* When the embedding parameter h goes from 0 to 1, $z(x,t;h)$ goes from the initial approximation to the exact solution as,

$$w(x,t) = \lim_{h\to 1} z(x,t;h) = z_0 + z_1 + z_2 + ...,$$

This gives the solutions of the above equations by using initial conditions as $z_0(x,t) = x + \ln t.$ and,

$z_1(x,t)=0$. And similarly the other components will be founded as, $z_k(x,t)=0;\quad k\ge 2$. Which will provide the solution as, $w(x,t)=x+\ln t$. Which is the same solution as it was calculated in *Adomian Decomposition Method*.

PROBLEM 5. 3 Consider the following PDE.

$$w_{tt}-w_{xx}+w^2=6xt(x^2-t^2)+x^6t^6, \tag{65}$$

with initial conditions as,

$$w(x,0)=0,\ w_t(x,0)=0. \tag{66}$$

We construct a Homotopy which satisfies,

$$(1-h)\Im[z(x,t;h)-w_0)]+h[\Im z(x,t;h)+z^2-z_{xx}]=6xt(x^2-t^2)+x^6t^6,$$

$$\Im z(x,t;h)-\Im w_0+h\Im w_0+h[z^2-z_{xx}]=6xt(x^2-t^2)+x^6t^6, \tag{67}$$

With initial approximation as,

$$w_0(x,t)=w(x,0)=0. \tag{68}$$

Suppose the solution of the form,

$$z(x,t;h)=z_0(x,t)+hz_1(x,t)+h^2z_2(x,t)+..., \tag{69}$$

Then we have,

$$\Im[z_0+hz_1+h^2z_2+...]-\Im z_0+h\Im z_0+h[(z_0{}^2+2hz_0z_1+h^2z_1{}^2)-(z_{0xx}+hz_{1xx}+h^2z_{2xx}+...)]= 6xt(x^2-t^2)+x^6t^6, \tag{70}$$

With the initial approximations as,

$$z(x,0;h)=z_0(x,0)+hz_1(x,0)+h^2z_2(x,0)+...,=0 \tag{71}$$

$$z_t(x,0;h)=z_{0t}(x,0)+hz_{1t}(x,0)+h^2z_{2t}(x,0)+...,=0 \tag{72}$$

Now equating the terms with equal power of h, the following set of equations are obtained,

$$\Im z_0=6xt(x^2-t^2)+x^6t^6;z_0(x,0)=0,z_{0t}(x,0)=0, \tag{73}$$

$$\Im z_1+[z_0{}^2-z_{0xx}]=0;z_1(x,0)=0,z_{1t}(x,0)=0 \tag{74}$$

$$\Im z_2+[2z_0z_1-z_{1xx}]=0;z_2(x,0)=0,z_{2t}(x,0)=0 \tag{75}$$

And so on. Again, note that we are getting the same components of the series solution as we have calculated for the *Adomian polynomials*, in *Adomian decomposition method.* When the embedding parameter h goes from 0 to 1, $z(x,t;h)$ goes from the initial approximation to the exact solution as,

$$w(x,t)=\lim_{h\to 1}z(x,t;h)=z_0+z_1+z_2+...,$$

So we have that,

$$z_0\left(x,t\right)=x^3t^3. \tag{76}$$

And similarly other components can also be calculated being vanished. Which will provide the solution as, $w(x,t)=x^3t^3.$ Which is the same solution as it was calculated in *Adomian Decomposition Method*.

CONCLUSIONS

In this study, approximated solutions for some non linear problems are calculated by *Adomian Decomposition Method* (ADM) and *Homotopy Perturbation Method* (HPM) and then the results are compared. It is analyzed that in *Adomian Decomposition Method*, first *Adomian polynomials* are calculated which is a bit difficult task and time consuming process and the fact that the *Homotopy Perturbation Method* (HPM) solves nonlinear problems without using *Adomian's polynomials* is a clear advantage of this technique over the *Adomian Decomposition Method* (ADM). The comparative study between these two methods shows that the results obtained by using *Homotopy Perturbation Method* (HPM) with a special convex constructed Homotopy is almost equivalent to the results obtained by using *Adomian Decomposition Method* (ADM) for these types of non linear problems.

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